DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA

UNEDITED ROUGH DRAFT TRANSLATION

INTERACTION BETWEEN SMALL PERTURBATIONS WITH DISCONTIN-
UITIES AND THE STABILITY OF SHOCK WAVES IN MAGNETOHYDRODYNAMICS

BY: V. M. Kontorovich

English Pages: 11

SOURCE: Russian Book, Voprosy Magnitnoy Gidrodinamiki Plazmy,
Akademiya Nauk Latviyskoy SSR, Riga, 1959, pp. 117-125

S/169-61-0-7-80/104

Date 14 May 19 63

P

INTERACTION BETWEEN SMALL PERTURBATIONS WITH DISCONTINUITIES
AND THE STABILITY OF SHOCK WAVES IN MAGNETOHYDRODYNAMICS

V. M. Kontorovich, Khar'kov

1. Annotation

We will examine the incidence of a plane monochromatic wave on a surface of discontinuity in magnetohydrodynamics. A simple geometric method is proposed for plotting divergent (reflected and refracted) waves which occur in this case (V. M. Kontorovich [1]).

Based on the results obtained, we will examine the problem of the resistance of shock waves to splitting (A. I. Akhiyezer [2]) with respect to oblique perturbations. It is shown that the number of waves diverging from the discontinuity does not depend on the angle of incidence. We stress the need to classify waves into incident and divergent waves by group velocity and not by phase velocity since at certain angles of incidence their projections onto the normal to the discontinuity have different signs. With a classification based on phase velocity we would have the paradox of unstable shock waves in magnetohydrodynamics.

The geometric method of plotting waves diverging from the discontinuity can be applied in simplified form to the interface of two

media in a compressible fluid, and also to a tangential discontinuity.

We examined the occurrence of surface waves in a compressible fluid upon interaction between perturbations and shock waves. It is shown in particular that surface waves arising upon reflection of waves from the interface and on their passage through the boundary in a compressible fluid (R. N. Roberts [3]) are a limiting case of that branch of magneto-acoustic waves which approach ordinary sound and disappear in an infinite fluid when $S \to \infty$.

We observed the peculiarities of the Doppler effect during interaction between small perturbations and shock waves. We found frequency drifts of the arising perturbations relative to the frequency of the incident perturbation (in the system of coordinates in which the shock wave moves) The frequency conversion can be quite large due to a change to perturbations of a type different than an incident wave. Measurements of the frequency shift theoretically allow us to restore the shock wave parameters.

General expressions are obtained for the coefficients of reflection from shock waves and for the passage through them. The final formulas are given in the simplest case of normal incidence to a perpendicular shock wave.

Equating the determinant of the system for amplitudes to zero, we obtain a characteristic equation which should be the basis of the investigation of stability relative to small wavelike perturbations of the surface of discontinuity. The region of spontaneous radiation of sound (resonance region) corresponds to real values of frequency and of wave vectors. The characteristic equation for fast and slow shock waves and also for rotational discontinuities have a different form.

2. Designations

The wave vector

$$k = (q \cot \alpha, q), \quad \varkappa = \frac{k}{k}. \tag{1}$$

The normal to the undisturbed surface of discontinuity $n = (1,0,0)$

The projection of the wave vector onto the plane of discontinuity

$$q = (0, q \sin \beta, q \cos \beta). \tag{2}$$

The angle of incidence

$$\alpha = \arccos \frac{k_x}{k} = \arccot \frac{k_x}{q}. \tag{3}$$

The angle between the direction of the wave vector and the magnetic field is θ.

The frequency in the frame of reference 1, where the surface of the shock wave is at rest is ω, whereas in the frame of reference where the fluid is at rest, it is ω_0. The moduli of phase velocity of perturbation in the direction $\varkappa$ for fast and slow magneto-acoustic waves and Alfven waves [4] are respectively:

$$V_+(\varkappa), \; V_-(\varkappa), \; V_A(\varkappa). \tag{4}$$

The curve expressing the dependence of phase velocity on angle of incidence in the plane of incidence, i.e., for $\beta = $ const, is

$$V_+(\alpha), \; V_-(\alpha), \; V_A(\alpha) \tag{5}$$

(in polar coordinates).

The half-space in front of the shock wave is $\overline{\text{II}}$; behind the shock wave, it is II. The magnitudes pertaining to $\overline{\text{II}}$ are designated by $\overline{A}$, and those to II by A. The jump in magnitude at the discontinuity is

$$(A) = A - \overline{A}. \tag{6}$$

The angle between the total velocity of perturbation $\pm V \cdot \varkappa + r_x n$ (with regard to drift) and the velocity of this perturbation $\pm V \cdot \varkappa$ relative to the fluid at rest is designated by ψ:

$$\cot \psi = \frac{\omega - q \cdot V}{q \, r_x} = \frac{r_x \cos \alpha \pm V(\alpha)}{r_x \sin \alpha},$$ (7)

where

    V = fluid velocity;

    s = speed of sound;

    p = pressure;

    H = magnetic fluid;

    $u = \frac{H}{\sqrt{4\pi\rho}}$;

    ρ = density;

    σ = entropy;

    w = heat function.

The group velocity of perturbation (in the frame of reference where the discontinuity surface is at rest)

$$V^{gr} = \frac{\partial \omega}{\partial k}, \quad V_x^{gr} = r_x \pm \left[V \cdot \cos \alpha - \frac{\partial V}{\partial \alpha} \cdot \sin \alpha \right],$$ (8)

where ψ_m and α_m are the angle values at which $\frac{\partial \psi}{\partial \alpha} = 0$ (on the $V+$ curve)

Ω_0 and Ω' are the frequencies radiated by the generator and received by the receiver which is fixed relative to $\overline{\Pi}$ (relative to the fluid in front of the shock wave); the frequency drift is

$$\Delta = \frac{\Omega_0 - \Omega'}{\Omega_0}.$$ (9)

The expression for the perturbed discontinuity is

$$x = \eta \exp i \, (q \, r - \omega \, t).$$ (10)

The system of equations for amplitudes of divergent waves and of the oscillation amplitude of discontinuity $\eta (i\eta = \delta A_0^{(p)})$ is

$$\sum_{i=0}^{6} T_{ji}\,\delta A_i^{(r)} = \sum_{i=1}^{6} U_{ji}\,\delta A_i^{(m)}, \quad j=1,2,\dots 7.$$

(11)

The index j indicates the number of the boundary equation. Coefficients T_{ji} and U_{ji} are found by expanding the arbitrary perturbation to the sum of fundamental waves :

$$\delta v = a_{i,i}^{\pm}\,\delta p_{\pm(i)} + c_{i,(i)}\,\delta H_{(i)};$$
$$\delta H = b_{i,i}^{\pm}\,\delta p_{\pm(i)} + d_{i,(i)}\,\delta H_{(i)}.$$

(12)

Here $\delta p_{\pm}$ is the pressure amplitude in the $v_{\pm}$ wave, δH is the z-component of the magnetic field in the Alfven wave. The index (1) has the values of 1 and 2 and indicates whether the given wave is with the current (1) or against the current (2) (if we change to a normal incidence). In (12) is the summation with respect to $\pm$ and (i):

$$a = \frac{\omega_0}{\rho\,k^2 s^2}\cdot\frac{\omega_0^2\,k - k^2\,(K\,u)\,u}{\omega_0^2 - (k\,u)^2};$$
$$b = \frac{\omega_0^2}{\rho\,k^2 s^2}\cdot\frac{k^2\,H - (k\,H)\,k}{\omega_0^2 - (k\,u)^2};$$
$$c = \mp\frac{1}{\sqrt{4\pi\rho}}\cdot\frac{[k,\,H]}{[k,\,H]_z}, \quad d = \frac{[k,\,H]}{[k,\,H]_z}.$$

(13)

If the boundary equations on the discontinuity are reduced to the form

$$T_{j}\cdot i_{\eta} + \{T_{j}\,\delta\sigma + P_j\,\delta p + R_j\,\delta v + Q_j\,\delta H\} = 0,$$

(14)

then the coefficients of equation (11) will be expressed by the combination of magnitudes

$$(_j^+)_i \equiv P_j + R_j\,a_{i,i}^{\pm} + Q_j b_{i,i}^{\pm};$$
$$|_j|_i \equiv R_j c_{(i)} + Q_j\cdot d_{(i)}.$$

(15)

From the continuity of mass flow ($j = 1$)

$$T_{10} = \{\rho(\omega - q\cdot v)\}, \quad T_{11} = r\,r_{z_0}.$$
$$P_1 = r_z/s^2, \quad R_1 = \rho\cdot a, \quad Q_1 = 0.$$

(16)

From the continuity of energy flow ($j = 2$)

$$T_{20} = \left\{ \rho \left(\frac{r^2}{2} + w + \frac{H^2}{4\pi\rho} \right) (\omega - q \cdot v) + \rho r_x^2 \omega + \frac{(v \cdot H)(q \cdot H)}{4\pi} \right\};$$

$$T_{21} = r_x \left[r \left(\frac{r^2}{2} + w \right) + \rho T \right];$$

$$P_2 = r_x \left[1 + \frac{1}{s^2} \left(\frac{r^2}{2} + w \right) \right];$$ (17)

$$R_2 = \rho r_x v - \frac{H_x}{4\pi} \cdot H + \rho \left(\frac{r^2}{2} + w + \frac{H^2}{4\pi\rho} \right) n;$$

$$Q_2 = \frac{r_x}{2\pi} H - \frac{H_x}{4\pi} \cdot v - \frac{(v \cdot H)}{4\pi} \cdot n.$$

The continuity of momentum flow ($j = 3,4,5$) yields

$$T_{30} = -2\rho r_x q \cdot (v), \quad T_{31} = r_x^2 r;$$

$$P_3 = 1 + r_x^2 / s^2, \quad R_3 = 2\rho r_x \cdot n, \quad Q_3 = \frac{H}{4\pi};$$ (18)

$$T_{40} = \left\{ \rho r_x^2 k_y + \rho r_y (\omega - q \cdot v) + \frac{H_y}{4\pi} (q \cdot H) \right\}, \quad T_{41} = r r_x r_y;$$

$$P_4 = \frac{r_x r_y}{s^2}, \quad R_4 = (\rho r_y, \rho r_x, 0), \quad Q_4 = \left(-\frac{H_y}{4\pi}, -\frac{H_x}{4\pi}, 0 \right);$$ (19)

$$T_{50} = \left\{ \rho r_x^2 k_z + \rho r_z (\omega - q \cdot v) + \frac{H_z}{4\pi} (q \cdot H) \right\}, \quad T_{51} = r r_x r_z;$$

$$P_5 = \frac{r_x r_z}{s^2}, \quad R_5 = (\rho r_z, 0, \rho r_x), \quad Q_5 = \left(-\frac{H_z}{4\pi}, 0, -\frac{H_x}{4\pi} \right).$$ (20)

From the continuity of tangential components of the electrical field ($j = 6,7$)

$$T_{60} = \{ H_y (\omega - q \cdot v) + r_y (q \cdot H) \}, \quad T_{61} = 0;$$

$$P_6 = 0, \quad R_6 = (H_y, -H_x, 0), \quad Q_6 = (-r_y, r_x, 0);$$ (21)

$$T_{70} = \{ H_z (\omega - q \cdot v) + r_z (q \cdot H) \}, \quad T_{71} = 0;$$

$$P_7 = 0, \quad R_7 = (H_z, 0, -H_x), \quad Q_7 = (-r_z, 0, r_x).$$ (22)

The eighth equation ($j = 8$, the continuity of the normal component of the magnetic field) upon perturbations of type (10) follows from the two preceding equations.

3. Results

We will present a number of results from this examination. The law of reflection has the form

$$\psi^{(n)} = \psi^{(a)}.$$ (23)

According to expression (7), the ends of the phase velocity vectors for all reflected and incident waves satisfying expression (23), lie on one circumference (ψ-circumference) passing through the ends of the vector $v_x \cdot n$.
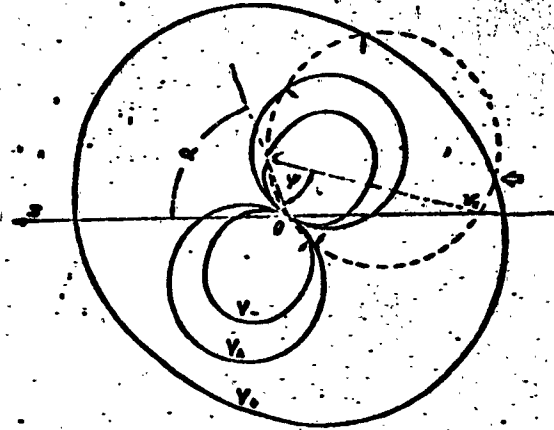

Fig. 1. Plotting of phase velocity vectors of reflected waves from a rapid inclined shock wave ($u/u = 0.4$, $\frac{\pi}{4}$, $\cos\theta = \frac{3}{4}\sin\epsilon + \frac{1}{2}\cos\epsilon$) for the medium behind the shock wave. The arrow indicates the points of intersection of the ψ-circumference with curves $V(\alpha)$. These points are the ends of the phase velocity vectors of the incident waves ($\frac{\partial t}{\partial s} < 0$, thick arrow) and of reflected wave ($\frac{\partial t}{\partial s} > 0$).

For slow shock waves and magnetohydrodynamic waves and for media in front of a shock wave a similar plotting (ψ-circumference) is necessary.

This enables us, by means of a cross section of surfaces $V + (\varkappa)$ $V_A(\varkappa)$, and $V - (\varkappa)$ with a plane of incidence, to construct the reflected waves with respect to a given shock wave; the intersection of ψ-circumference with $V(\alpha)$ curves will yield the ends of phase velocity vectors of the unknown waves.

Differentiating expression (7), we obtain:

$$\frac{\partial \psi}{\partial \alpha} = \frac{\sin^2 \psi}{r_2 \sin^2 \alpha} \cdot v_x^m.$$ (24)

whence it is obvious that the separation into incident and divergent waves can be done with respect to the symbol $\frac{\partial \psi}{\partial \alpha}$, which is determined from the same diagram.

The law of refraction is

$$(r_2 \cot \psi) + \frac{q}{q}(v) = 0.$$ (25)

If $q \cdot (v) = 0$, then expression (25) becomes "the law of tangents"

$$\tan \psi / \tan \overline{\psi} = r_x / \overline{r}_2.$$ (26)

In order to find all of the divergent waves, we must plot the $\overline{\psi}$ and ψ-circumferences, where ψ and $\overline{\psi}$ are related by condition (25), and must separate all waves diverging from discontinuity with respect to the symbol $\frac{\partial \psi}{\partial \alpha}$. For fast waves divergent waves do not occur in II and it is sufficient to plot only the ψ-circumference.*

When $v_x \to 0$, ψ-circumference at the limit occupies a position tangential to the normal (x-axis) at the origin of coordinates. Plotting of the reflected and refracted waves is as before. Since when $v_x \to 0$ formally $\psi \to 0$, then in place of the derivative $\partial \psi / \partial \alpha$ we can introduce a derivative (having the opposite sign) from the diameter of the periphery $\frac{\partial D}{\partial \alpha}$ and, by this value, classify the waves as incident and divergent.

The transition to an incompressible fluid results in that $l_+^2 \sim s^2 \to \infty$, a $k^2 = \frac{\omega^2}{l_+^2} \to 0$. For this, however, there remains in II the

_____

*These problems and also the occurrence of surface waves on passage through and reflection from a shock wave and the role of phase and group velocities have been examined by us in detail [1].

solution $k(+ iq, q)$, and in $\overline{\Pi} = \overline{k}(- iq, q)$, representing a surface wave. Therefore (ku) is finite when $s \rightarrow \infty$,

$$Q^+ \rightarrow \frac{k}{\rho \omega_0};$$

$$b^+ \rightarrow -\frac{(k\,H)\,k}{\rho \omega_0^2}. \tag{27}$$

The expression $r^2 \rightarrow (ku)^2 - \frac{u^4}{s^2} \cos^2 \Theta \sin^2 \Theta$, and the slow sound at the limit gives the Alfven baric wave

$$\frac{u}{H} b^- = \mp a^- = -\frac{1}{\rho u} \frac{\left[x, \left(\frac{u}{u}, x \right) \right]}{\sin^2 \Theta}. \tag{28}$$

The formulas for frequency drifts (9) owing to the Doppler effect and as a result of the conversion of some forms of perturbations to other forms for normal incidence to shock waves were presented in our article [1]. The frequency measurements can be used for the study of shock waves. On the other hand, the effect of change of frequency with simultaneous (generally speaking not small) change of amplitudes both toward a decrease and toward an increase is a new mechanism of frequency conversion with simultaneous amplification (attenuation) of the signal . This pertains, of course, to low frequencies, where magnetohydrodynamics is applicable.

We will introduce the coefficients of passage for a normal incidence of a signal to a perpendicular shock wave. For the fast shock wave we have:

$$T_{j0} \cdot i_1 + T_{j1} \delta \sigma + T_{j2} \delta p_{-(1)} + T_{j3} \delta H_{(1)} + T_{j4} \delta p_{+(1)} +$$
$$+ T_{j5} \delta p_{-(2)} + T_{j6} \delta H_{(2)} = \sum U_{ji} \delta A_i^{(m)};$$
$$T_{j2} = (\bar{j})_1, \quad T_{j3} = l_j l_1, \quad T_{j4} = (j)_1, \quad T_{j5} = (j)_2;$$
$$T_{j6} = l_j l_2, \quad U_{j1} = -(j)_2;$$
$$U_{ji} = T_{ji}, \quad k = 1, \dots 6, \quad U_{j4} = -U_{j1}.$$

The coefficients of passage for incidence of the $\overline{V}+$ wave

$$\frac{\delta p_{+(i)}}{\delta \bar{p}_{+(i)}} = \frac{\bar{M} + \bar{N}}{N(M+N)} \cdot \frac{\bar{M} - lN^2(\bar{M}+\bar{N})}{1 - lN(M+N)};\qquad(29)$$

$$\frac{\delta p_{-(i)}}{\delta p_{+(i)}} = A^2 \cdot \frac{\bar{M} + \bar{N}}{2M^2 N} \cdot \frac{M(\bar{M}+\bar{N}) - \bar{N}(M+N)}{1 - l(M+N)N};\qquad(30)$$

For incidence of the $\overline{V}_-$ wave

$$\frac{\delta p_{+(i)}}{\delta \bar{p}_{-(i)}} = \frac{\bar{M}}{MN(M+N)} \cdot \frac{\left(\bar{M}M - \bar{N}^2 \frac{\bar{s}}{s}\right) - lMM\left(N^2 - \bar{N}^2 \frac{\bar{s}^2}{s^2}\right)}{1 - l(M+N)N}.\qquad(31)$$

Here $M = r/s, \quad N = V_{i}/s, \quad A = u/s, \quad 1 + A^2 = N^2,$
$l = pT/rr(r) = c_p/\beta(p), \quad r = \left(\frac{\partial p}{\partial \sigma}\right)_p, \quad \beta = \left(\frac{\partial}{\partial T} \frac{1}{\rho}\right)_p.$

On changing to ordinary hydrodynamics ($N \to 1$, $A \to 0$) we obtain from expression (29) the coefficient of sound passage through a shock wave [5].

REFERENCES

1. <u>V. M. Kontorovich</u>.   ZhETF, 35, 5 (11), 1216, 1958.

2. <u>A. I. Akhiyezer, G. Ya. Lyubarskiy and P. V. Polovin</u>.   ZhETF, 35, 3, 731, 1958.

3. <u>P. H. Roberts</u>.   Astr. J., 121, 4, 720, 1955.

4. <u>S. I. Syrovatskiy</u>.   UFN, 62, 3, 247, 1957.

5. <u>V. M. Kontorovich</u>.   ZhETF, 33, 6 (11) , 1527, 1957.

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies	MAJOR AIR COMMANDS	Nr. Copies
		AFSC	
HEADQUARTERS USAF		SCFDD	1
		DDC	25
AFCIN-3D2	1	TDBTL	5
ARL (ARB)	1	TDEDP	5
		AEDC (AEY)	1
		SSD (SSF)	2
OTHER AGENCIES		AFFTC (FTY)	1
		AFWL (WLF)	1
CIA	1	ASD (ASYIM)	2
NSA	6		
DIA	9		
AID	2		
OTS	2		
AEC	2		
PWS	1		
NASA	1		
ARMY (FSTC)	3		
NAVY	3		
NAFEC	1		
AFCRL (CRXLR)	1		
RAND	1		